Data Modeling and Data Analysis in Simulation Credibility Evaluation of Autonomous Underwater Vehicles

Xiaojun Guo, Zhiping Huang, Shaojing Su, Yunxiao Lv

College of Intelligence Science and Technology,
National University of Defense Technology,
Changsha, Hunan, China
jeanakin@163.com; 3421436@qq.com; susj-5@163.com; jeanakin@126.com

Abstract-As an important tool for exploring and defending the ocean, autonomous underwater vehicles (AUVs) play an irreplaceable role. With the help of simulation models, the R&D test cycle of AUV equipment can be accelerated, but the simulation credibility assessment of AUVs faces many challenges: uncertainty, emergence and nonlinearity. This paper starts from the credibility evaluation of the simulation model of AUVs. Based on small-sample judgment criterion, Bayesian Sequential Mess Test (SMT) that makes full use of prior knowledge is proposed for the credibility evaluation of static parameters. For the reliability evaluation of the dynamic simulation model, the NARX steady-state response algorithm and the prior-based identification are used to evaluate the reliability of the dynamic simulation model. The application performance of the data analysis method in the credibility evaluation of AUVs is analyzed.

Keywords-autonomous underwater vehicle(AUV); credibility evaluation; Bayesian sequential mess test; non-linear autoregressive model with exogenous inputs (NARX) steady-state response algorithm

I. INTRODUCTION

As an important tool for exploring and defending the ocean, autonomous underwater vehicles (AUVs) play an irreplaceable role, and simulation credibility evaluation of AUVs has the characteristics of uncertainty, emergence and nonlinearity. Whether the reliability of the simulation model can be effectively evaluated determines the maturity of the AUV technology. AUV systems not only has complex test environmental conditions, but also the complex and randomly changing hydrological conditions under water, the control law based on fluid dynamics, and the limited underwater communication methods have all added certain difficulties to the AUV experiment[1-2]. The number of tests for the evaluation of model parameters and indicators is very limited (tens of times at most). Due to the lack of support from actual flight test data, it is difficult to evaluate the reliability of the simulation model. This problem also constrains a considerable part of AUV M&S research. The reference model used in the verification of the AUV simulation model and the relevant data of the model to be verified are the sink information under a certain channel transmission[3-4]. The uncertainty of the system [1]error and the inaccuracy of the experimental data observation are the main causes of the uncertainty.

An approach for mixture for symmetric distributions was proposed. They focused on the two-component mixture and develop d a Bayesian model using parametric priors for the location parameters and a nonparametric prior based on Dirichlet process [5]. As the similar method handling with small-size sample and high precision, the application of bootstrap is no less than Bayesian method, and also achieved surprising improvement [6]. For the simulation modelling and reliability evaluation of complex large systems, many methods based on expert systems have been proposed[7-8].

In this paper, starting from the reliability evaluation of the simulation model of AUV, firstly, for the reliability evaluation of the static simulation model, a small sample judgment criterion based on precision measurement is given; The Bayesian SMT identification test that makes full use of prior knowledge is used for the reliability evaluation of static parameters; secondly, for the reliability evaluation of the dynamic simulation model, the NARX steady-state response method and the prior-based identification are used to evaluate the reliability of the dynamic simulation model. The structural parameters of the model are identified, thereby transforming the reliability evaluation of the dynamic model into the performance evaluation of the static parameter distribution. Finally, the application performance of the data analysis method in the reliability evaluation of the complex large system is analyzed.

II. DATA ANALYTICS METHODS IN CREDIBILITY EVALUATION OF AUV STATIC SIMULATION MODELS

When analyzing the relevant performance of the small sample test, the traditional statistical method based on classical frequentism has been unable to reasonably explain the test results under the background of the small sample due to its limitations. Most studies use Bayesian statistical methods when the sample size cannot meet the precision requirements of specific applications, but they do not give an exact conceptual definition of small samples [9].

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A. Definition of small sample

Based on the application of statistical inference in different contexts, it can be inferred that the size of the sample is judged based on the application. In view of the differences between classical frequency statistics and Bayesian statistics, the related concepts of point estimation, interval estimation in classical frequency statistics are used to help explain the definition of sample size.

[Definition 1] A random variable has a density distribution function $f(X)$ .Suppose its variance is σ , the precision required by the application is δ_0 , the point estimation of a certain mathematical characteristic parameter is $\hat{\theta}$, then the precision of this estimation is $\sigma(\hat{\theta}) = \frac{\sigma}{\sqrt{n}}$, n is the sample capacity, then

(1) n satisfies $n > (1/\lambda)ceil(\sigma^2/\delta_0^2), 0 < \lambda < 1$ is the large-sample size of the significance degree $1/\lambda$of the mathematical feature parameter point estimation under the distribution.

(2) n satisfies $n < (1/\eta)ceil(\sigma^2/\delta_0^2), 1 < \eta$ is the small-sample size of the significance degree $\eta$of the mathematical feature parameter point estimation under the distribution.

[Definition 2] A random variable has a density distribution function$f(X)$. When the confidence level is$1-\alpha$, the interval estimation of a certain mathematical characteristic parameter is $[\hat{\theta}(X)-\delta, \hat{\theta}(X)+\delta]$ The required precision is δ_0, $\delta = g(f(X), n)$, n is sample capacity, then

(1) n satisfies $\delta < \lambda\delta_0, 0 < \lambda < 1$ is the large-sample size with a significance degree of $1/\lambda$ in the estimation of the mathematical feature parameter interval under the distribution;

(2) n satisfies$\delta > \eta\delta_0, \eta > 1$ is the small sample size of the significance level η of the mathematical feature parameter interval estimation under the distribution.

B. Bayesian Sequential Mess Test

During World War II, in order to meet the needs of military acceptance work, A. Wald proposed a sequential inspection method, sequential probability ratio test (SPRT). and proved that in all test classes where the probability of making two types of errors does not exceed a given sum of α and β , the average SPRT required test sample (ie, the test sample size) is minimal. Two goals can be achieved with sequential testing as below:

(1) It is expected to reduce the number of tests under the same identification accuracy requirements. The method constructs a buffer region between the rejection region and the acceptance region, avoiding drastically different decisions based on the success or failure of a single trial.

(2) The sampling times can be adjusted according to the current inspection or estimation effect, so that the sample size can be appropriately selected, so that the obtained estimation has a predetermined accuracy; or under a given sampling cost, the risk can be reduced.

Compared with the traditional method, the SPRT method has been greatly improved, and the improvement in reducing the test sample size is significant, but this method does not take into account the prior information, so that the historical test data or empirical data are not fully utilized, and the test sample size is still large. In fact, the model assumptions are usually biased, that is, the robustness of the SPRT method, and the optimality of SPRT is only established under certain hypothetical models.

In the case of fully considering the prior information, based on SPRT, the sequential posterior odd test (SPOT) method is proposed. Given two types of risk upper thresholds (denoted as α_N,β_N), for the truncated test scheme T_N, if these probabilities of the determined decision scheme are within the allowable range, the SPOT truncated scheme $T_N$is judged to be desirable. The solution of the SPOT truncation scheme is transformed into the analysis of the relationship between the decision threshold C and the sample size N and the two types of risks. For the specific application background, the computer-aided method can be used for fitting and solving.

Aiming at the shortcomings of SPRT, the Sequential Mess Test (SMT) method is constructed for the testing scheme of simple hypotheses against simple hypotheses, and it has been proved that it can effectively reduce the test sample size under the condition of equivalent risk. The idea of this method is to split the original two-alternative hypothesis testing problem into multiple groups of hypothesis testing problems under the condition of given two-alternative hypothesis test values p_0, $p_1$and two types of risk upper limit valuesα, β. Taking the SMT hypothesis test with one point inserted as an example, $p_2 \in (p_1, p_0)$ the original SPRT hypothesis test is divided into the following two groups of hypothesis test problems:

$$H_{01}: p = p_2,\ H_{11}: p = p_1$$
$$H_{02}: p = p_0,\ H_{12}: p = p_2$$

For the two sets of hypothesis tests, the SPRT method is used to test them respectively, so that the finite value can be obtained when the algorithm is stopped. Figure 1 depicts an SMT scheme that inserts a point. It can be seen from Figure 1 that the sample size required by this method has an upper bound, when the population distribution tested is a binomial distribution, the upper bound is the intersection of two straight lines.

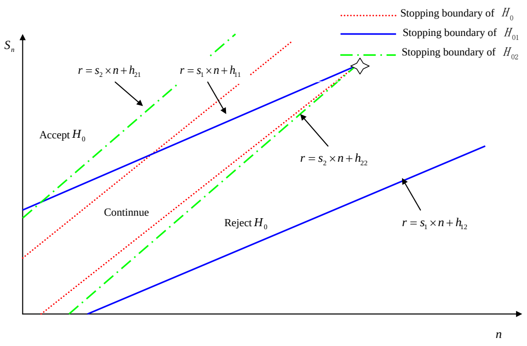

Figure 1. SMT solution for inserting a checkpoint

The minimum sample size of the truncated SMT scheme is also much better than that of the traditional method, but the SMT algorithm that simply inserts multiple points has little improvement in the test effect. This paper combines the above two ideas and constructs a new inspection scheme, which may achieve better improvement effect, which is called the Bayes SMT method. The construction of Bayes SMT test needs to solve the following problems: a) acquisition, quantification and rationality test of prior information; b) Splitting of prior information; c) determination of the principle of Bayes SMT; d) optimal insertion point solution; e) Risk size; f) Truncated program design. As shown in Figure 2, the introduction of SMT test with prior information makes it possible to further reduce the test area, that is, to further reduce the test sample size.

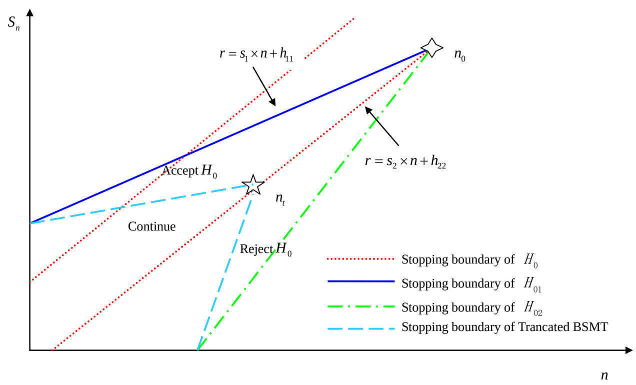

Figure 2. BSMT solution for inserting a checkpoint

Since the starting point of the design of the SMT scheme is the sequential test of simple hypotheses against simple hypotheses, this method is used for multiple hypothesis testing, but cannot well solve the testing problem with complex hypotheses. But in solving the problem of sequential testing of simple hypotheses, it can still better reduce the amount of calculation.

III. Data Modeling and Data Analysis Methods in the Credibility Evaluation of Dynamic Simulation Models

A. *Steady-state response method for NARX model*

In the early stage of nonlinear system identification theory, a general nonlinear regression model with exogenous variables (non-linear autoregressive model with exogenous inputs, NARX) was proposed. As a universal model of NARX, NARMAX (non-linear autoregressive moving average models with exogenous inputs) model basically covers almost all nonlinear models such as bilinear models, H-models, W-models, nonlinear time series models, ARMAX models, etc. As illustrated in Figure 3.

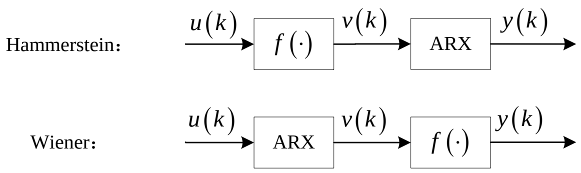

Figure 3. Block-connected nonlinear characterization of Hammerstein and Wiener models

The general NARX model can be expressed as

$$y(k) = F^{\ell}[y(k-1), \cdots, y(k-n_y), u(k-d), \cdots, u(k-n_u), e(k)] \quad (1)$$

Equation (1) can be expanded into a polynomial sum of nonlinearity in the interval $[1, \ell]$,The $(p+m)$th term includes a p-order $y(k-n_i)$, an m-order $u(k-n_i)$, and a multiple factor $c_{p,m}(n_1, \cdots, n_m)$, as shown in Equation (2).

$$y(k) = \sum_{m=0}^{\ell} \sum_{p=0}^{\ell-m} \sum_{n_1,n_m}^{n_y,n_u} c_{p,m}(n_1, \cdots, n_m) \prod_{i=1}^{p} y(k-n_i) \prod_{i=1}^{m} u(k-n_i) + e(k) \quad (2)$$

B. *NARX characterization of AUV motion models*

The nonlinear state equation of the motion model of the 6-DOF AUV can be expressed as

$$\begin{cases} \dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{y} = g(\boldsymbol{x}) \end{cases} \quad (3)$$

Where,

$\boldsymbol{x} = [v_x, v_y, v_z, w_x, w_y, w_z, x_b, y_b, z_b, \psi, \theta, \phi]^T$, $\boldsymbol{u} = f(v_x, v_y, v_z, w_x, w_y, w_z, \delta_E, \delta_R, \delta_D)$,

$$e_1 = m(v_y w_z - v_z w_y) + m x_G (w_y^2 + w_z^2) - m y_G w_x w_y - m z_G w_x w_z + \lambda_{22} - \lambda_{33} v_z w_y + \lambda_{26}(w_y^2 + w_z^2) - (mg - F_G)\sin\theta + Te + Rx$$

$$e_2 = m(v_z w_x - v_x w_z) + m y_G (w_x^2 + w_z^2) - m z_G w_y w_z - m x_G w_x w_y + \lambda_{33} v_z w_x - \lambda_{11} v_x w_z - \lambda_{26} w_x w_y - (mg - F_G)\cos\theta\cos\varphi + Ry$$

$$e_3 = m(v_x w_y - v_y w_x) + m z_G (w_x^2 + w_y^2) - m x_G w_x w_z - m y_G w_y w_z + \lambda_{11} v_x w_y - \lambda_{22} v_y w_x - \lambda_{26} w_x w_z + (mg - F_G)\cos\theta\sin\varphi + Rz$$

$$e_4 = (I_{yy} + \lambda_{55} - I_{zz} - \lambda_{66}) w_y w_z + m y_G z_G (w_y^2 - w_z^2) + m x_G z_G w_x w_y - m x_G y_G w_x w_z + m y_G (v_x w_y - v_y w_x) + m z_G (v_x w_z - v_z w_x) + m g y_G \cos\theta + m g z_G \cos\theta\cos\varphi + Mx$$

$$e_5=(I_{zz}+\lambda_{66}-I_{xx}-\lambda_{44})w_xw_z+mx_Gz_G(w_z^2-w_x^2)+mx_Gy_Gw_yw_z-my_Gz_Gw_xw_y-mgz_G\sin\theta$$
$$+my_G(v_yw_z-v_zw_y)+mx_G(v_yw_x-v_xw_y)+\lambda_{26}(v_yw_x-v_xw_y)+mgx_G\cos\theta\sin\varphi+My$$
$$e_6=(I_{xx}+\lambda_{44}-I_{yy}-\lambda_{55})w_xw_y+mx_Gy_G(w_x^2-w_y^2)+my_Gz_Gw_xw_z-mx_Gz_Gw_yw_z+mgy_G\sin\theta$$
$$+mx_G(v_zw_x-v_xw_z)+my_G(v_zw_y-v_yw_z)+\lambda_{26}(v_zw_x-v_xw_z)-mgx_G\cos\theta\cos\varphi+Mz$$

$\boldsymbol{E}=[e_1,e_2,e_3,e_4,e_5,e_6]^T$, $\boldsymbol{M}=\boldsymbol{M}_I+\boldsymbol{M}_A$, $[\dot{v}_x,\dot{v}_y,\dot{v}_z,\dot{w}_x,\dot{w}_y,\dot{w}_z]^T=\boldsymbol{M}^{-1}\boldsymbol{E}$,

$$\begin{bmatrix}\dot{x}_a\\ \dot{y}_b\\ \dot{z}_b\end{bmatrix}=\boldsymbol{C}_0^b\begin{bmatrix}v_x\\ v_y\\ v_z\end{bmatrix},$$

$$\begin{bmatrix}\dot{\psi}\\ \dot{\theta}\\ \dot{\phi}\end{bmatrix}=\begin{bmatrix}(w_y\cos\phi-w_z\sin\phi)/\cos\theta\\ w_y\sin\phi+w_z\cos\phi\\ w_x-(w_y\cos\phi-w_z\sin\phi)\tan\theta\end{bmatrix}$$

TABLE I. PARAMETERS AND DEFINITIONS IN AUV MOTION MODEL

Variable	Definitions
m	AUV quality (kg)
S	Characteristic area, generally take the largest cross-sectional area (m^2)
L	Feature length, generally take the total length of AUV (m)
v	AUV speed(m/s)
$\delta_E,\delta_R,\delta_D$	Horizontal rudder, straight rudder, differential rudder
ρ	The density of water, here $994kg/m^3$
$c_x(\cdot),c_y(\cdot),c_z(\cdot)$	Drag, lift, side force coefficients
$m_x(\cdot),m_y(\cdot),m_z(\cdot)$	Roll moment, yaw moment, pitch moment coefficient
x_G,y_G,z_G	Center of gravity backward shift, descent, side shift
I_{xx},I_{yy},I_{zz}	Inertia torque in X, Y, Z directions
λ_{ij}	Component coefficients for additional inertial forces and moments
α,β	Angle of attack, angle of sideslip
v_x,v_y,v_z	Velocity components in X, Y, Z directions
w_x,w_y,w_z	Angular velocity components in the X, Y, Z directions
x_b,y_b,z_b	The position component of the AUV in the ground coordinate system
ψ,θ,ϕ	Yaw angle, pitch angle, roll angle
D_t	AUV displacement
g	Gravitational acceleration, here take 9.81(m/s^2)

So far, the mechanism modeling of the functional relationship $\dot{\boldsymbol{x}}=f(\boldsymbol{x},\boldsymbol{u})$ has been completed. Taking the observation equation $\boldsymbol{y}=g(\boldsymbol{x})=\dot{\boldsymbol{x}}$, there is a discretized NARMAX state equation as

$$\boldsymbol{X}(k+1)=F^{\ell}(\boldsymbol{X}(k),\boldsymbol{U}(k),\boldsymbol{C}_0^b,Te,Rx,\cdots,Mx,\cdots)\quad(1.1)$$

For Equation (1), the function $f(\cdot)$ between the steady-state intermediate signal $v(k)$ and the output signal $y(k)$ is steady-state, and it is the gain of the ARX model. If the steady-state gain is adjusted to 1, that is $\bar{v}(k)=\bar{u}(k)$, it is the difference between the input and the output, the steady-state response function $f(\cdot)$. The function can be used to obtain the corresponding relationship of $\bar{y}\times\bar{u}=\bar{v}$, and a certain linear regression method can be used to obtain the function estimate $\bar{v}=g(\bar{y})$, which is an estimate on the application domain. Assuming that the model (shown in equation (2)) is excited by a constant input, then the steady-state response is

$$\bar{y}=y(k-1)=y(k-2)=\cdots=y(k-n_y),\bar{u}=u(k-1)=u(k-2)=\cdots=u(k-n_u)\quad(4)$$

Then Equation (2) can be rewritten as

$$y(k)=\sum_{m=0}^{\ell}\sum_{p=0}^{\ell-m}\sum_{n_1,n_m}^{n_y,n_u}c_{p,m}(n_1,\cdots,n_m)\,\bar{y}^p\bar{u}^m\quad(5)$$

This gives the model response at a specific input point.

C. NARX Model Steady-State Response Method for Grey Box Identification

The method of parameter identification using the steady-state response of the SISO-NARX model can be extended to multi-dimensional situations, and the same example is still used for analysis. This nonlinear system is the MISO-NARX system. The steady-state response parameter identification method of NARX is based on The SISO system proposes that the labels of variables in these methods are all defined based on SISO. When the identification of MISO-NARX is realized, the relevant labeling methods need to be improved. It can be known from the combination function $P_i(k)$,

$$y_0(k)=\sum_{i=1}^{12}d_if^{\ell}(v_x,v_y,v_z,w_x,w_y,w_z,\theta,\phi,\dot{v}_y,\dot{w}_x,\dot{w}_z,R_y)+d_0\quad(6)$$

In Equation (6), there are a total of 12 inputs, although there is no input in the form of $y_0=\dot{v}_x$ itself, but is a component of the multi-dimensional output

$\boldsymbol{Y}=[\dot{v}_x,\dot{v}_y,\dot{v}_z,\dot{w}_x,\dot{w}_y,\dot{w}_z,\dot{x}_b,\dot{y}_b,\dot{z}_b,\dot{\psi},\dot{\theta},\dot{\phi}]^T$,

$n_y=n_u=1$,so the nonlinear NARX relationship with the above 12 inputs needs to be considered when $u_i(k-1)=u_i(k)$.

From the test data without identification design, select the data that meets the conditions for steady-state response identification, that is, the steady-state response process. The 100 data samples used in the above two algorithms come from a direct flight and speed-up process. Therefore, except when $v_x(k-1)=v_x(k)=15.2408$ entering a steady state, all other observed variables are 0 or close to 0. At this time, there are

$$-\varepsilon<(mg-\rho D_tg)\left(1+\frac{\theta^2}{2}\right)\left(1+\frac{\phi^2}{2}\right)-\bar{R}_y<\varepsilon\quad(7)$$

The steady-state input referred to $\bar{R}_y$ in Equation (7), ε is the identification accuracy requirement, and the steady-state output of at this time is -310.82, which is why the identification parameter $mg - \rho D_t g$ always has a small identification variance no matter which identification algorithm is used. Figure 4.5 shows part of the time series of test data that can be used for identification. For the convenience of observation, the data has been transformed such as translation and compression. The identification carried out by Equation (7) is carried out by intercepting a meta-process or meta-process segment in the test. Searching for the segment that can be used for steady-state response identification in the whole process, the identification segment and identification inequality can be obtained, as shown in Figure 4.

As shown in Table II, the identification of the first two methods uses the sampling data of 0-40s, and the sampling period is 0.025s. It can be seen from the table that for this example, the effect of parameter identification is NARX steady-state corresponding method RFF-LS RPEA-BP. The forgetting factor algorithm of RFF-LS gives priority to new information to prevent parameter identification drift caused by data saturation, and makes full use of prior information and experimental collection of new information. The BP neural network recognizes the internal structure and parameters of the sample system through training, but for the model system whose structure is known, the accuracy of the parameter identification results is poor. The grey-box identification method based on the NARX steady-state response makes full use of the characteristics of the input-output relationship of the system's steady-state response in the NARX model. Although there is no special experimental design requirement for the belt identification system, a sufficient amount of steady-state response identification inequality is indispensable.

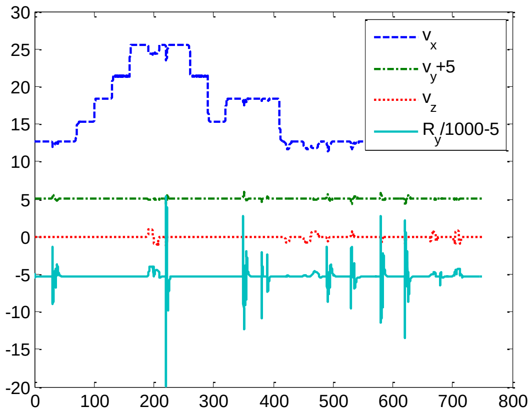

Figure 4. Partial identification data

TABLE II. THREE ALGORITHMS BASED ON THE SAME PRIOR CONSTRAINTS TO IDENTIFY THE FITTING ACCURACY OF PARAMETERS

Period	**Accumulation of identification errors**		
	RFF-LS	RPEA-BP	Steady state response method
0-40s	6.5254	14.5911	5.5376
0-200s	132.851	213.284	78.265
0-400s	416.4397	520.515	222.549
0-750s	814.675	1302.32	416.61

The parameters identified in different field tests are $\theta_i (i = 1,2,\cdots,N)$, where N is the number of field tests that can be used for grey box identification, and the model parameter based on mechanism modeling is θ_0, at this time, the reliability of the dynamic model based on grey box identification is transformed into a test of whether it follows the distribution, or whether it falls within the estimated mean interval with a certain confidence level. For θ_i, assuming that it obeys a normal distribution, estimate the Bootstrap BCa interval estimate under each confidence level. Then examine the placement points, and measure the credibility level of the virtual model (mechanism modeling) with confidence.

IV. CONCLUSION

Based on the test accuracy requirements, the criterion of sub-sample capacity traits is given, which provides a quantitative standard for the determination of sub-sample size; when conducting the sequential hypothesis testing of small sub-sample test index parameters, reference is made on the basis of SPRT, SPOT, and SMT. Bayes theory proposes the Bayes SMT test method, which can theoretically save the sample size based on the prior; a grey box parameter identification method for the NARX model based on prior and steady-state response is proposed. The identification results show that this method improves the parameter identification of the AUV equation of motion accuracy and identification efficiency. On the basis of grey box identification, using prior distribution information and modern statistical inference methods, the reliability assessment of dynamic models is transformed into the reliability assessment of static parameter models, which provides a method for credibility assessment of dynamic models with prior information. new way.

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